Template Example: Solution of the Heat Conduction in Solid-State Electronics by Integral Transforms

Prof. Daniel J. N. M. Chalhub (Presenter)
Lívia M. Corrêa

Dep. of Mech. Engineering – PPG-EM,
Universidade do Estado do Rio de Janeiro,
Rio de Janeiro - RJ, Brazil

COBEM – December 2017

Introduction

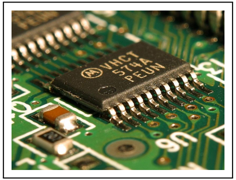
One of the biggest challenges on the design of modern electronic devices is the thermal control.

In this work, it is proposed a solution by Classical Integral Transform Technique(CITT) to solve a heat conduction problem on Solid-State Electronics (SSE) considering heat generation and external convection.

Solid State Electronics (SSE)

Solid-state electronics (SSE) are those circuits or devices built entirely from solid materials and in which the electrons, or other charge carriers, are encapsulated within the solid material.

Exemples: Transistors, Microprocessors, Random Access Memory (RAM), Itegrated Circuits (IC).



Substrate

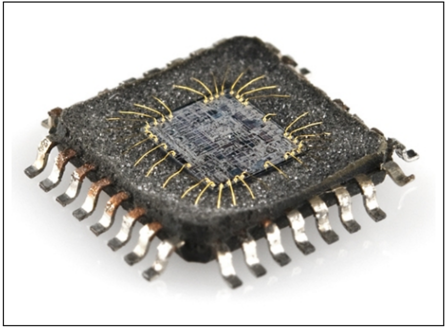
Substrate is a solid substance and serve as the foundation upon which electronic devices such as transistors, diodes, and especially Integrated Circuits (ICs) are encapsulated.

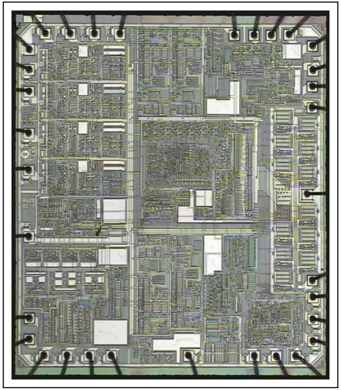
Example of substrate's materials:

- silicon
- silicon dioxide
- aluminum oxide
- sapphire

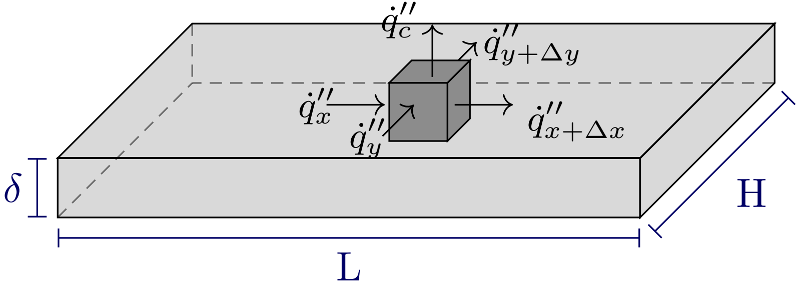
The advantage of this is the superior insulation between adjacent transistors.

Exemple of SSE: Integrated Circuit





Problem Description



Problem Formulation

The nondimensionalization of the problem leads to the following mathematical formulation:

$$\frac{\partial^2 \Theta}{\partial \xi^2} + \beta^2 \frac{\partial^2 \Theta}{\partial \eta^2} - (\mathrm{Bi}\gamma)\Theta = -G(\xi, \eta) \quad \text{for} \quad 0 \leq \xi \leq 1 \quad \text{and} \quad 0 \leq \eta \leq 1$$

$$\left.\frac{\partial \Theta}{\partial \xi}\right|_{\xi=0} = 0; \qquad \left.\frac{\partial \Theta}{\partial \eta}\right|_{\eta=0} = 0;$$

$$\left.\frac{\partial \Theta}{\partial \xi}\right|_{\xi=1} = 0; \qquad \left.\frac{\partial \Theta}{\partial \eta}\right|_{\eta=1} = 0;$$

Classical Integral Transform Technique

In order to solve the proposed problem, the Classical Integral Transform Technique (CITT) is applied, considering the appropriate Helmholtz Eigenvalue Problem in cartesian coordinates.

$$\Psi_n''(\eta) + \lambda_n^2 \Psi_n(\eta) = 0$$
$$\Psi_n'(0) = 0; \quad \Psi_n'(1) = 0.$$

where $\Psi(\eta)$ are the eigenfunctions and λ_n are the eigenvalues. For this particular problem, the case where $\lambda = 0$ also exists.
For $\lambda = 0$, the solution of the eigenvalue problem is given by:

$$\Psi_0(\eta) = 1; \qquad \lambda_0 = 0;$$

And for $\lambda > 0$:

$$\Psi_n(\eta) = \cos(\lambda_n \eta); \qquad \lambda_n = n\pi, \quad \text{for} \quad n = 1, 2, 3, \ldots$$

Classical Integral Transform Technique

To apply the CITT, the transformation pair is defined.

Transformation Pair

$$\text{Transformation} \quad \Rightarrow \quad \bar{\Theta}_n(\xi) = \int_0^1 \Theta \Psi_n(\eta) \mathrm{d}\eta$$

$$\text{Inversion} \quad \Rightarrow \quad \Theta = \sum_{n=0}^{\infty} \frac{\bar{\Theta}_n(\xi)\Psi_n(\eta)}{N_n}$$

N_n is the norm and is defined as:

$$N_n = \int_0^1 \Psi_n^2 \mathrm{d}\eta$$

Classical Integral Transform Technique

The dimensionless diferential equation is written again, multiplied by Ψ_n and integrated in the domain for η.

$$\int_0^1 \frac{\partial^2\Theta}{\partial\xi^2}\Psi_n d\eta + \beta^2 \int_0^1 \frac{\partial^2\Theta}{\partial\eta^2}\Psi_n d\eta - \mathrm{Bi}\gamma \int_0^1 \Theta\Psi_n d\eta = -\int_0^1 G\Psi_n d\eta$$

The transformed equation is obtained and admits an analytical solution.

For $\lambda > 0$:

$$\bar{\Theta}_n'' + (-\beta^2\lambda_n^2 - \mathrm{Bi}\gamma)\bar{\Theta}_n = -\bar{G}_n(\xi)$$

$$\bar{\Theta}_n'(0) = 0; \quad \bar{\Theta}_n'(1) = 0$$

where

$$\bar{G}_n(\xi) = \int_0^1 G(\xi,\eta)\Psi_n(\eta)\mathrm{d}\eta$$

For $\lambda = 0$:

$$\bar{\Theta}_0'' - (\mathrm{Bi}\gamma)\bar{\Theta}_0 = -\bar{G}_0(\xi)$$

$$\bar{\Theta}_0'(0) = 0; \quad \bar{\Theta}_0'(1) = 0$$

where

$$\bar{G}_0(\xi) = \int_0^1 G(\xi,\eta)\mathrm{d}\eta$$

Classical Integral Transform Technique

The transformed equations admit a analytical solutions.

In order to obtain the final temperature field, the inversion formula must be utilized and the summation must be truncated to a finite value ($n_{\max}$).

$$\text{Inversion} \quad \Rightarrow \quad \Theta = \sum_{n=0}^{n_{\max}} \frac{\bar{\Theta}_n(\xi)\Psi_n(\eta)}{N_n}$$

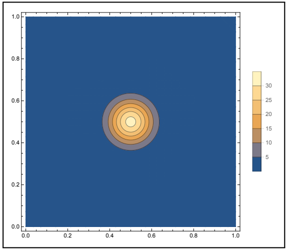
Heat Generation

$$G(\xi, \eta) = \frac{G_e}{\sigma^2 \pi} \exp\left[-\frac{(\xi - \xi_0)^2 + (\eta - \eta_0)^2}{\sigma^2} \right]$$

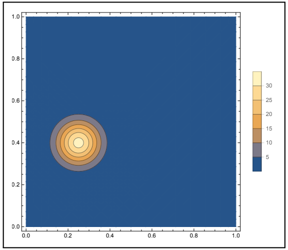
For this current work, two cases are considered:

- Centered HG where $\xi_0 = 1/2$ and $\eta_0 = 1/2$
- Off-Centered HG where $\xi_0 = 1/4$ and $\eta_0 = 2/5$

For both cases $G_e = 1$ and $\sigma = 0.1$.



(f) Centered HG



(g) Off-Centered HG

Results

Table: $\Theta(\xi, \eta)$ convergence solved for Centered HG by CITT

	Biγ=1 & β=1				Biγ=0.1 & β=1			
n_{max}	Θ(0.2,0.4)	Θ(0.5,0.5)	Θ(0.6, 0.8)	Θ(0.9,0.9)	Θ(0.2, 0.4)	Θ(0.5, 0.5)	Θ(0.6,0.8)	Θ(0.9,0.9)
1	0.981222	1.05643	1.03739	0.966733	9.98087	10.0575	10.0382	9.96593
5	1.00221	1.18948	0.997708	0.954075	**10.0025**	10.1922	9.99803	**9.95274**
10	**1.00213**	1.20298	1.00209	**0.954083**	10.0025	10.2058	10.0024	9.95274
20	1.00213	**1.20323**	**1.00213**	0.954083	10.0025	**10.2060**	**10.0025**	9.95274
30	1.00213	1.20323	1.00213	0.954083	10.0026	10.2060	10.0025	9.95274
	Biγ=1 & β=0.8				Biγ=0.1 & β=0.8			
n_{max}	Θ(0.2,0.4)	Θ(0.5,0.5)	Θ(0.6,0.8)	Θ(0.9,0.9)	Θ(0.2,0.4)	Θ(0.5,0.5)	Θ(0.6,0.8)	Θ(0.9,0.9)
1	0.981222	1.05643	1.03739	0.966733	9.98087	10.0575	10.0382	9.96593
5	1.01980	1.23495	0.980675	0.940388	10.0212	10.2395	9.98041	9.93805
10	**1.01957**	1.25429	0.987398	**0.940418**	**10.0210**	10.2589	9.98715	**9.93808**
20	1.01957	**1.25467**	**0.987461**	0.940418	10.0210	**10.2593**	**9.98722**	9.93808
30	1.01957	1.25467	0.987461	0.940418	10.0210	10.2593	9.98722	9.93808
	Biγ=1 & β=0.5				Biγ=0.1 & β=0.5			
n_{max}	Θ(0.2,0.4)	Θ(0.5,0.5)	Θ(0.6,0.8)	Θ(0.9,0.9)	Θ(0.2,0.4)	Θ(0.5,0.5)	Θ(0.6,0.8)	Θ(0.9,0.9)
1	0.981222	1.05643	1.03739	0.966733	9.98087	10.0575	10.0382	9.96593
5	1.09885	1.38536	0.921228	0.874113	10.1105	10.4039	9.91626	9.86204
10	1.09731	1.42441	0.936699	0.874431	**10.1090**	10.4432	9.93187	**9.86236**
20	**1.09730**	**1.42526**	**0.936864**	**0.874429**	10.1090	**10.4441**	**9.93203**	9.86236
30	1.09730	1.42526	0.936864	0.874429	10.1090	10.4441	9.93203	9.86236

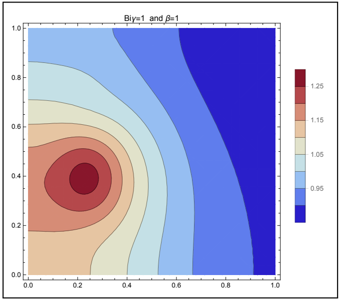
Results

Table: $\Theta(\xi, \eta)$ convergence solved for Off-Centered HG by CITT

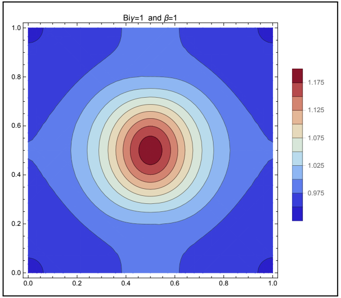
	Biγ=1 & β=1				Biγ=0.1 & β=1			
n_{max}	Θ(0.2,0.4)	Θ(0.5,0.5)	Θ(0.6,0.8)	Θ(0.9,0.9)	Θ(0.2,0.4)	Θ(0.5,0.5)	Θ(0.6,0.8)	Θ(0.9,0.9)
1	1.14595	0.991937	0.920318	0.865087	10.1555	9.99006	9.91129	9.84928
5	1.26434	1.02009	0.914914	**0.863013**	10.2754	10.0190	9.90570	**9.84707**
10	1.27061	**1.01987**	**0.91489**	0.863013	10.2817	**10.0188**	**9.90568**	9.84707
20	**1.27083**	1.01987	0.91489	0.863013	**10.2819**	10.0188	9.90568	9.84707
30	1.27083	1.01987	0.91489	0.863013	10.2819	10.0188	9.90568	9.84707
	Biγ=1 & β=0.8				Biγ=0.1 & β=0.8			
n_{max}	Θ(0.2,0.4)	Θ(0.5,0.5)	Θ(0.6,0.8)	Θ(0.9,0.9)	Θ(0.2,0.4)	Θ(0.5,0.5)	Θ(0.6,0.8)	Θ(0.9,0.9)
1	1.1564	0.991937	0.900772	0.846859	10.1682	9.99006	9.88666	9.82559
5	1.32297	1.04002	0.890144	**0.840922**	10.3380	10.0401	9.87552	**9.81914**
10	1.33221	**1.03950**	**0.890058**	0.840922	10.3473	**10.0396**	**9.87544**	9.81914
20	**1.33253**	1.03950	0.890058	0.840922	**10.3476**	10.0396	9.87544	9.81914
30	1.33253	1.03950	0.890058	0.840922	10.3476	10.0396	9.87544	9.81914
	Biγ=1 & β=0.5				Biγ=0.1 & β=0.5			
n_{max}	Θ(0.2,0.4)	Θ(0.5,0.5)	Θ(0.6,0.8)	Θ(0.9,0.9)	Θ(0.2,0.4)	Θ(0.5,0.5)	Θ(0.6,0.8)	Θ(0.9, 0.9)
1	1.18823	0.991937	0.830465	0.772371	10.2157	9.99006	9.77686	9.70579
5	1.52152	1.12367	0.794684	0.738850	10.5654	10.1341	9.73726	9.66629
10	1.54154	1.12104	0.793933	**0.738828**	10.5855	10.1315	9.73650	**9.66627**
20	1.54228	**1.12102**	**0.793928**	0.738828	**10.5862**	**10.1314**	**9.73649**	9.66627
30	**1.54229**	1.12102	0.793928	0.738828	10.5862	10.1314	9.73649	9.66627

Results

Figure: Illustration of the solution by CITT with the different heat generation positions with the parameters of Biγ=1 and β=1.



(a) Off-Centered HG case



(b) Centered HG case

Conclusions

- The solution is fully analytical
- CITT had a great performance to obtain high accuracy with very few terms in the solution summation.
- This is a preliminary work and for near future works, one can propose: Use of the complete Heat Generation and variation of the heat conductivity.

Acknowledgments

Thank You

Acknowledgments:

CAPES CNPq FAPERJ

PPG-EM Programa de Pós-graduação em Engenharia Mecânica

UERJ Universidade do Estado do Rio de Janeiro

Lists - Examples

- Point A
- Point B
    - part 1
    - part 2
- Point C

I. Point A
II. Point B
    i. part 1
    ii. part 2
III. Point C

API Application Programming Interface

LAN Local Area Network

ASCII American Standard Code for Information Interchange

Block Texts - Examples

Block Title

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod

Block Title

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor

Definition

A prime number is a number that...

Example

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua.

Including Code - Examples

```
\begin{gather}
\frac{\partial^2 T}{\partial x^2} +
\frac{\partial^2 T}{\partial y^2}
 = 0
\end{gather}
```